Eur. Phys. J. B **24**, 463–468 (2001)

THE EUROPEAN PHYSICAL JOURNAL B

EDP Sciences

Hopping conductivity and activated transport in $In_xGa_{1-x}As$ quantum wells

A. Urbina[1,a], C. Díaz–Paniagua[2], A.F. Braña[2], and F. Batallán[2]

[1] Departamento de Electrónica, Tecnología de Computadoras y Proyectos, Universidad Politécnica de Cartagena, C/Doctor Fleming s/n, 30202 Murcia, Spain
[2] Instituto de Ciencia de Materiales de Madrid, Consejo Superior de Investigaciones Científicas, Cantoblanco, 28049 Madrid, Spain

Received 6 June 2001 and Received in final form 16 October 2001

Abstract. We present measurements of the diagonal R_{xx} and off-diagonal R_{xy} magnetoresistance under quantum Hall conditions on several high electron mobility transistors (HEMT) based on $In_xGa_{1-x}As$ quantum wells. From the magnetoresistance tensor we obtain the longitudinal conductivity σ_{xx}. We study the transport mechanisms near the σ_{xx} minima at temperatures ranging between 2 K and 35 K; activated transport is the dominant mechanism for temperatures above 7 K while variable range hopping conductivity is significant for lower temperatures. We show that electron-electron correlations should be taken into account to explain the conductivity *vs* temperature behaviour below 5 K. Finally, we study the behaviour of the localization length as a function of Landau level filling and obtain a critical exponent $\gamma = 3.45 \pm 0.15$.

PACS. 72.80.Ey III-V and II-VI semiconductors – 73.43.Qt Magnetoresistance – 72.20.Ee Mobility edges; hopping transport

1 Introduction

The integer quantum Hall effect in a disordered two-dimensional (2D) electron gas provides an experimental system to study localization to delocalization transitions. Early experimental studies [1] and theoretical results [2] found that the width of the transition region shrinks as the temperature goes to zero according to a power law of the form

$$\Delta B, (\delta\rho_{xy}/\delta B)^{-1}_{\max} \propto T^{\kappa}, \tag{1}$$

with an universal exponent $\kappa = 0.42 \pm 0.04$, which was obtained by Wei *et al.* [3,4]. The universality of κ is still controversial and it has been a point of discussion. Experimental measurements on $In_xGa_{1-x}As$ heterostructures [5,6] and on Si-MOSFETs (metal-oxide-semiconductor field-effect transistors) [7] led to the conclusion that κ is not universal showing a dependence with carrier mobility and doping densities, taking values ranging between $0.28 \pm 0.06 < \kappa < 0.81 \pm 0.03$. This critical exponent is closely related to the behaviour of the localization length, ξ, near the Fermi level, which is described by the expression:

$$\xi(\nu) = \xi_0 |\nu - \nu_0|^{-\gamma}, \tag{2}$$

where ν is the Landau level filling and ν_0 is the filling where the localization length diverges, *i.e.* the mobility edge. The maximum localization (ξ_0 minimum) is obtained at even Landau level fillings, when σ_{xx} is a minimum and the plateau of the integer quantum Hall effect is well defined. Most of the studies on the scaling behaviour of integer quantum Hall effect have been devoted to the transition between adjacent plateaus. In this paper we focus our interest on the minimum of Shubnikov-de Haas oscillations, where localization is maximum and the 2D electron gas provides an experimental system where transport mechanism between localized states can be studied.

When electron states are localized and the low temperature limit is considered, the only possible mechanism of electronic transport is variable–range hopping. Theoretical calculations of the temperature dependence of σ_{xx} including the effects of Coulomb interactions performed by Efros and Shklovskii [8] led to a modification of Mott's law of variable range hopping [9]. They found that the conductivity, when considering the existence of the Coulomb gap, is of the form,

$$\sigma_{xx} = \sigma_0 \exp\{-(T_0/T)^{1/2}\}, \tag{3}$$

with the exponent 1/2 independent of the dimensionality of the system. T_0 is given by the expression:

$$T_0 = \beta \frac{e^2}{\epsilon k_{\mathrm{B}} \xi}, \tag{4}$$

[a] e-mail: aurbina@upct.es

where ξ denotes the localization length of the electrons and β is a numerical coefficient that depends on dimensionality. Using the usual percolation theory for hopping transport one obtains $\beta \approx 6.2$ for 2D systems. Evidence of the importance of electronic correlations due to exchange in uncompensated p-type insulating Si:B when a magnetic field is applied were early reported [10]. Since then, several experimental measurements on a wide variety of samples imply that β should have a smaller value than predicted by Efros and Shklovskii [11–13]. The quantum Hall conditions of a 2D electron gas is nowadays the best experimental system to perform magnetotransport measurements at low temperatures and much effort have been devoted to relate variable range hopping with the scaling behaviour observed in such systems. Polyakov and Shklovskii [14] explained the broadening of the conductivity peak in terms of variable range hopping,relating the critical exponent γ of equation (2) to the variable range hopping parameter T_0. They obtained $\gamma \approx 2.3$. A detailed calculation of the effect of screening of the Coulomb interaction on the conductivity in the quantum Hall conditions was performed later [15]. This screening is provided by the metallic gate evaporated parallel to the plane of the 2D electron gas present in the transistor used in measurements and should be taken into account in order to explain experimental results obtained on such structures. The κ exponent of equation (1) turned out to be equal to $1/2\gamma$, observation of such dependence will prove that Coulomb interactions play an important role for the conductivity in the minima of the Shubnikov-de Haas oscillations. More recently, Pérez–Garrido *et al.* [16] performed detailed computer calculations taking full account of many-body effects of Coulomb interactions on strongly localized electron systems. They showed that correlations in the motion of electrons cannot be neglected in a transport theory in the Coulomb gap, the β value obtained being an order of magnitude lower than the reported by Efros and Shklovskii in two-dimensional systems.

For higher temperatures than previously considered, hopping conductivity can be damped by activated transport. The activation-type conductivity does exists when temperature is high enough to bring electrons from the localized states to the mobility edge of the next empty Landau level. The contribution to the conductivity of such activated transport is given by the following equation,

$$\sigma_{xx} = \sigma_0 \exp(-E_\mathrm{g}/2k_\mathrm{B}T), \tag{5}$$

where E_g is the energy gap between the Fermi level and the mobility edge. When the Fermi level is placed on localized states at an integer Landau level filling (where localized states are supposed to be [17]), the energy gap is half the energy between adjacent Landau levels (in an ideal case without considering the broadening provided by impurities and imperfections). If we move the Fermi level along the density of localized states towards the next Landau level, filling empty localized states the energy gap reduces linearly. In a 2D electron gas existing in transistor systems, this is achieved by applying a variable gate voltage to the semiconductor heterostructure.

Table 1. Summary of characterization results of samples S1 and S2 at zero gate voltage: electron density, effective mass, transport lifetime τ_0 and quantum life time τ_q.

	$n_\mathrm{e}(10^{16}\ \mathrm{m}^{-2})$	m^*/m_0	τ_0(ps)	τ_q(ps)
S1	1.70 ± 0.01	0.066 ± 0.002	0.87 ± 0.03	0.11 ± 0.01
S2	1.50 ± 0.01	0.063 ± 0.004	0.80 ± 0.05	0.20 ± 0.01

For temperature dependent magnetotransport measurements in the minima of Shubnikov-de Haas oscillations, both transport mechanisms should be considered. At lower temperatures, variable range hopping will be the fundamental contribution to transport between localized states while for higher temperatures, approximately above 5 K, activated transport will become more important damping the hopping behaviour. At low magnetic fields, these measurements cannot be accomplished because Landau levels are very close to each other and localized states are too close to extended ones, in this case hopping conductivity does not play any role. Experimental measurements should be performed for high magnetic fields so Landau levels would be clearly resolved.

2 Samples and measurements

We have performed magnetotransport measurements on two different samples, called S1 and S2 in this paper. Both are pseudomorphic high electron mobility transistors (HEMT) based on $\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As}$ channels with different kind of doping layers: S1 sample is delta doped and S2 sample is modulated doped. Both samples were grown by Molecular Beam Epitaxy (MBE) on semiinsulating (100) GaAs substrates. The S1 sample has a 120 Å thick $\mathrm{In}_{0.3}\mathrm{Ga}_{0.7}\mathrm{As}$ channel followed by an $\mathrm{Al}_{0.3}\mathrm{Ga}_{0.7}\mathrm{As}$ barrier in which is embedded a doping Si delta layer $N_\mathrm{D} = 2.45 \times 10^{12}\ \mathrm{cm}^{-2}$, a 50 Å thick spacer is placed over the well, the sample is finally covered by a 200 Å GaAs caplayer. The S2 sample has a 130 Å thick $\mathrm{In}_{0.18}\mathrm{Ga}_{0.82}\mathrm{As}$ channel followed by 190 Å thick $\mathrm{Al}_{0.2}\mathrm{Ga}_{0.8}\mathrm{As}$ barrier which is homogeneously Si doped $N_\mathrm{D} = 3 \times 10^{18}\ \mathrm{cm}^{-3}$, a 20 Å thick spacer is placed over the well. In order to perform the measurements, Hall bar shapes were photolithographied on the samples, with a distance of 325 μm between longitudinal resistance probes and a 3.5 length-to-width ratio. Ti-Al top gates were evaporated on the Hall bars to modulate charge into the channel using a gate voltage. Both samples have been well characterized [18], a summary of characterization parameters is given in Table 1. For sample S1, the delta doped one, an oscillatory behaviour of effective mass depending on Landau level filling under strong magnetic fields have been reported [19]. In this paper, we present results for a fixed magnetic field at different Landau level fillings as will be explained below. A well defined value for the effective mass can be considered for every empirical calculation.

We have measured, using the four-point technique, the diagonal (R_{xx}) and off-diagonal (R_{xy}) components of the

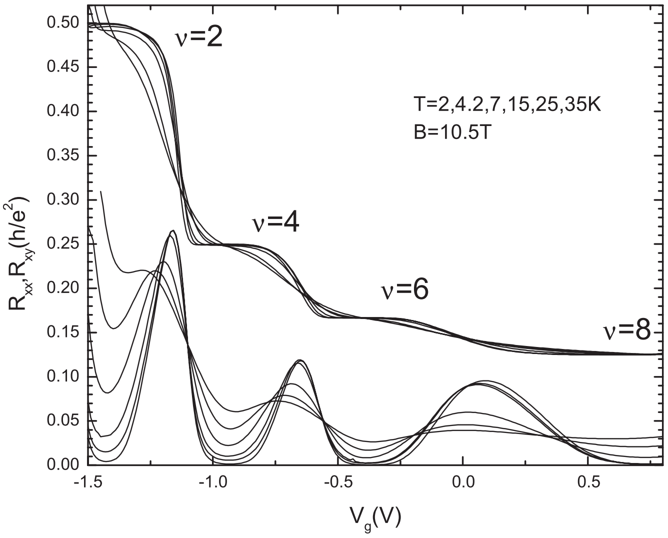

Fig. 1. Diagonal and off-diagonal magnetoresistance measurements performed on S1 sample at different temperatures as function of applied gate voltage at a fixed magnetic field $B = 10.5$ T. R_{xx} shows the Shubnikov-de Haas oscillations and R_{xy} shows the integer quantum Hall plateaus for $\nu = 2, 4, 6, 8$ Landau level fillings.

transverse magnetoresistance tensor. The magnetic field was applied perpendicular to the two dimensional electron gas of the heterostructure. The maximum magnetic field was 12 T. The temperature was highly stabilized for each measurement in a wide range going from 2 K to 35 K. When the temperature is stabilized, we introduce a continuous current of 10 μA which flows between both extremes of the Hall bar. V_{xx} was measured between longitudinal probes and V_{xy} between transversal probes, the magnetoresistance tensor is directly obtained from this measurements. For each temperature two kinds of measurements were performed: a magnetic field sweeping for a fixed gate voltage and a gate voltage sweeping for a fixed magnetic field (in this case we used 10.5 T for measurements on S1 sample and 7.87 T on S2 sample, this value is selected in order to study the lowest Landau levels). We show in Figure 1 the magnetoresistance *vs* gate voltage measurements performed on S1 at $B = 10.5$ T for some temperature values. We observe the integer quantum Hall plateaus at the expected values for even Landau level fillings (for $i = 2$, 4, 6 and 8) and the corresponding Shubnikov-de Haas oscillations on the diagonal component. The amplitude of the oscillations decreases with the temperature and it is possible to select particular values for a determined Landau level filling (at a specific gate voltage value) to study the behaviour of the magnetoresistance as a function of temperature. The advantage of this procedure is that temperature is always very exactly determined avoiding gradient effects along the sample because each temperature value is stabilized for a long time using a PID intelligent temperature controller. It guarantees a fixed temperature during the complete measurement. Afterwards we can take magnetoresistance values for every gate voltage (equivalent to every Landau level filling) to obtain the dependence with temperature in the range from 2 K to 35 K avoiding temperature displacements and temperature gradients along the sample.

3 Results

From both the diagonal and off-diagonal components of the magnetoresistance tensor we can obtain by inverting such tensor the components of conductivity, in particular σ_{xx}, the diagonal component, which is often called longitudinal conductivity:

$$\sigma_{xx} = \frac{R_{xx}/LW}{(R_{xx}/LW)^2 + R_{xy}^2}, \tag{6}$$

where LW is the length to width ratio of the samples. Our experimental results are very dense data files so we have R_{xx} and R_{xy} data for every temperature, magnetic field and gate voltage and we can obtain, using equation (6), indirect experimental values of the longitudinal conductivity. Results are obtained for all measurements in the temperature range going from $T = 2$ K to $T = 35$ K. The conductivity σ_{xx} are obtained for different Landau level fillings at a fixed magnetic field (for S1 sample at $B = 10.5$ T and for S2 sample at $B = 7.87$ T). The structure of Landau levels remains fixed at a constant magnetic field and we "move" the Fermi level across this structure when we change the gate voltage. Two special points at every Landau level should be mentioned. The first one is the minima of the oscillations, it corresponds to the half extent of the quantum Hall plateaus in the off-diagonal conductivity, Fermi level are then anchored on the localized states which populates this special point of the density of states where magnetic induced localization is maximum. The second interesting point is the crossing of all the plots at the mobility edge, beyond which, the Fermi level is located on extended states and the quantum Hall plateus remains no longer quantified, furthermore, they show the transition from one plateau to the next one.

We have studied in detail the conductivity behaviour as a function of temperature for a great set of Landau level fillings from the minima of the oscillation to the mobility edge. For every filling, given by a fixed gate voltage value, we select all the different temperature points to construct the behaviour of the conductivity with the temperature in the range from 2 K to 35 K. This can be seen in Figure 2 for S1 sample, where σ_{xx} is plotted as a function of $T^{-1/2}$ for twenty-two different gate voltages ranging from $V_{\rm g} = -0.94$ V to $V_{\rm g} = -0.72$ V. The first gate voltage corresponds to a Fermi level placed on localized states at the bottom of the oscillation (at integer $\nu = 4$ Landau level filling) and the last one to the next mobility edge of empty extended states. Straight lines showing the fitting for the lower temperature range (below 5 K) are superimposed to the experimental points. From this kind of plot (σ_{xx} at logarithmic scale as a function of inverse of temperature or as a function of square root of inverse of temperature) we can obtain by linear fitting the transport

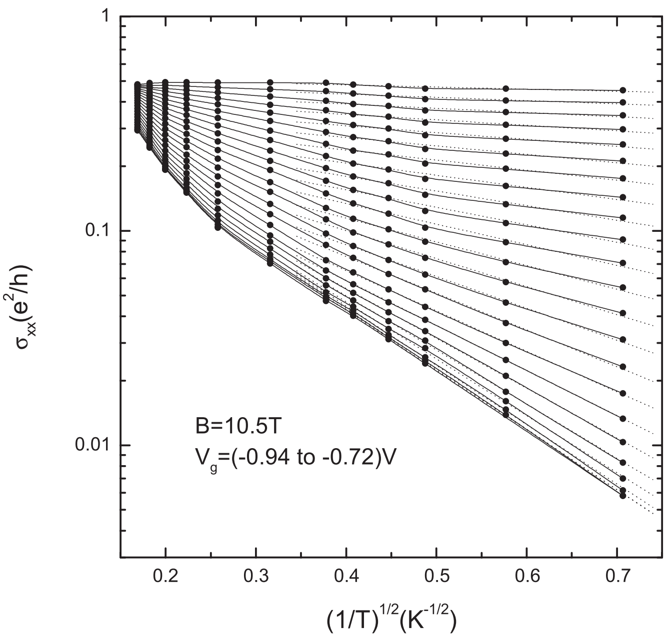

Fig. 2. Diagonal conductivity of sample S1 as a function of the square root of the inverse of the temperature for different Landau level fillings ranging from the minima of the Shubnikov-de Haas oscillation which corresponds to $\nu = 4$ Landau level to the next mobility edge. Applied gate voltage ranges from -0.94 V to -0.72 in steps of 0.05 V. We also show the linear fitting (in logarithmic scale) for the lower temperature range of the twenty two plots.

Table 2. Summary of experimental results of samples S1 and S2. For every sample at a fixed magnetic field we show the transport parameters for variable range hopping: T_0 obtained experimentally and β deduced from the former, and for activated transport, the energy gap between Fermi level and mobility edge, E_g. (Errors are lower than 5%.)

	Magnetic field (T)	Landau level	T_0(K)	β	E_g(meV)
		2	39.5	0.25	2.16
S1	10.5	4	40.8	0.26	2.20
		6	35.4	0.23	2.53
		4	16.9	0.13	1.21
S2	7.87	6	16.3	0.12	1.23
		8	16.2	0.12	1.42

parameters of both mechanisms which are playing a role: variable range hopping for lower temperatures (below 5 K, fitting the $1/T^{1/2}$ plot), and activated transport for higher temperatures (up to 35 K, fitting the $1/T$ plot). This can be done for every Landau level filling from the minima of the oscillation to the mobility edge.

In Table 2 we show the results for the minima of the Shubnikov-de Haas oscillations at three integer Landau level fillings for both samples. From the higher temperature portion of the plots we obtained the activation energies, say, the energy gap from the Fermi level to the mobility edge, called E_g in equation (5). It corresponds (for $\sigma_{xx,\mathrm{min}}$) to half distance between the maximum of the Landau level peaks of the density of states. From the lower temperature portion we obtain the variable range hopping experimental parameter T_0 of equation (3); from this value, the parameter β can be calculated using equation (4), which is supposed to apply when the localization is maximum. This happens when the Fermi level is at the bottom of the Shubnikov-de Haas oscillation, where we can consider that the localization length is the shortest one and takes for its value the magnetic length given by $\xi_0 \approx l = (\hbar/eB)^{1/2}$, where B is the applied magnetic field. In equation (4) we have used the dielectric constant for the material of the quantum well, which is the channel where electrons move. It is usual to take $\varepsilon(\mathrm{In}_x\mathrm{Ga}_{1-x}\mathrm{As}) = (1-x)\varepsilon(\mathrm{GaAs})+x\varepsilon(\mathrm{InAs})$ when such material is a ternary alloy; from the values for GaAs and InAs we obtain for S1: $\varepsilon_{S1} = 13.77$ and for S2: $\varepsilon_{S2} = 13.54$. At the end, we have a relation between the experimentally measured T_0 and the parameter β, which is also shown in Table 2. The values obtained are an order of magnitude lower than predicted by one-electron theory. They are much closer to numerical calculations which take into account many-body effects [16].

Finally, we discuss the transition from localized to delocalized states. It is observed when the Fermi level is moved across the Landau level structure by varying the gate voltage applied to the semiconductor heterostructure for a given magnetic field. The transition happens when the Fermi level moves from the minima of Shubnikov-de Haas oscillations, which corresponds to well defined integer quantum Hall plateaus and where localization is maximum (minimum localization length), to the mobility edge where extended states begin to be occupied. We have studied in detail the case of the transition from Landau level $\nu = 4$ to the next mobility edge on S1 sample. We obtained the transport parameters for different Landau level fillings ranging from the minima of the oscillation at $V_g = -0.94$ V to the mobility edge at $V_g = -0.72$ V. The activated transport parameters gives the energy gap for every Landau level filling showing as expected a linear dependence with the gate voltage, as can be seen in Figure 3: E_g changes linearly from his maximum value to zero. We can state that $\Delta V_g \propto \Delta\nu$. From the variable range hopping parameter T_0, obtained at every Landau level filling, we can deduce the localization length ξ taking for the $\nu = 4$ level the value $\beta = 0.26$ previously obtained. The results are shown on Figure 4 where we plot logarithmically ξ as a function of the Landau level filling, obtaining clearly a behaviour described by equation (2). We obtain the critical exponent of equation (2) from the linear fitting shown in Figure 4. The value obtained for the critical exponent is $\gamma = 3.45 \pm 0.15$.

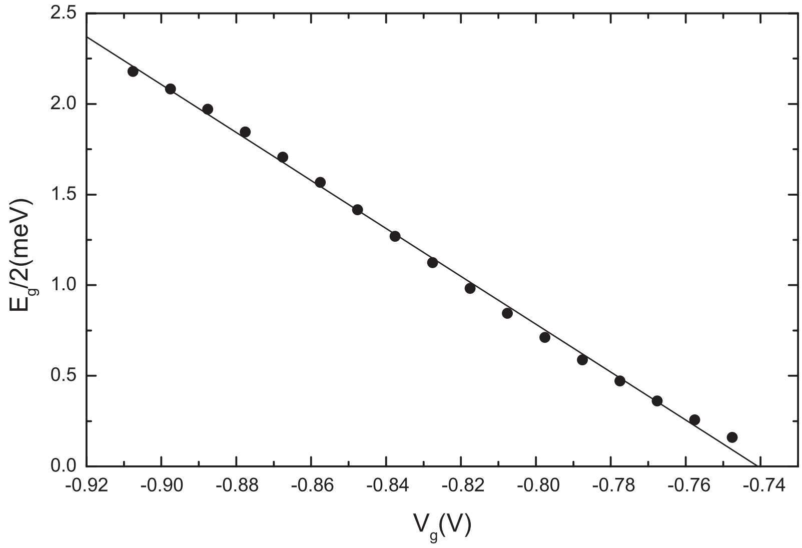

Fig. 3. Activation energy deduced from transport parameters for different Landau level fillings from $\nu = 4$ Landau level to the next mobility edge for S1 sample. The expected linear behaviour with applied gate voltage is clearly observed.

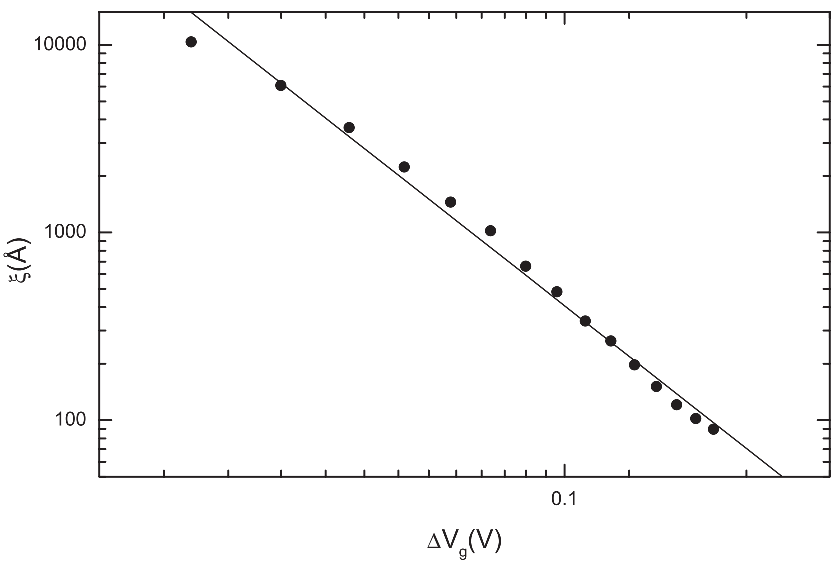

Fig. 4. Critical behaviour of the localization length ξ as a function of Landau level filling as it grows from the minimum length (maximum localization) at $\nu = 4$ Landau level to the next mobility edge. It can be observed how ξ diverges when Fermi level approximates to the mobility edge.

4 Conclusion

We have performed magnetoresistance measurements on a two–dimensional electron gas confined in the quantum well of an $In_xGa_{1-x}As$ high electron mobility transistor. We studied in detail the temperature dependence of conductivity at the minima of Shubnikov-de Haas oscillations where localized states are contributing to electronic transport. Two transport mechanisms play an important role, but at very low temperature, variable range hopping is the only mechanism. We obtain the parameter $\beta = 0.26 \pm 0.02$ which is an order of magnitude lower than the value predicted by one-electron theory, reinforcing the opinion that electron correlations play an important role on variable range hopping conductivity of such systems. At higher temperatures activated transport turns out to be the dominant mechanism and damps the hopping contribution. Localization to delocalization transition have been studied and a parameter $\gamma = 3.45 \pm 0.15$ is obtained.

We would like to acknowledge financial support from CICYT project number MAT 95-0771. We would like to thank to Susana Fernández de Ávila and F. González Sanz for sample preparation.

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